



香港中文大學

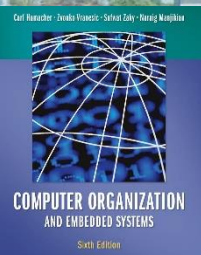
The Chinese University of Hong Kong

*CSCI2510 Computer Organization*

# Lecture 02: Number and Character Representation

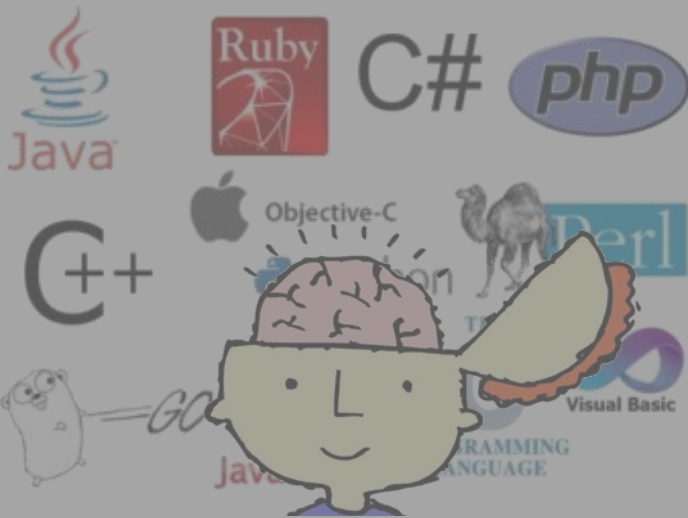
**Ming-Chang YANG**

[mcyang@cse.cuhk.edu.hk](mailto:mcyang@cse.cuhk.edu.hk)



Reading: Chap. 1.4~1.5, 9.7~9.8

# Recall: How to talk to the computer?



**High-level Language**

Easy for programmer to understand

Human understandable English words

**Language Translation**

**Machine Language**

The computer's own language

Binary numbers (All 1s and 0s)



- Number Representation
  - Number Systems
  - Integers
    - Unsigned Integer
    - Signed Integer
    - Arithmetic Operations
  - Floating-Point Numbers
    - Unsigned Binary Fraction
    - Floating-Point Number Representation
    - Arithmetic Operations
- Character Representation
  - ASCII

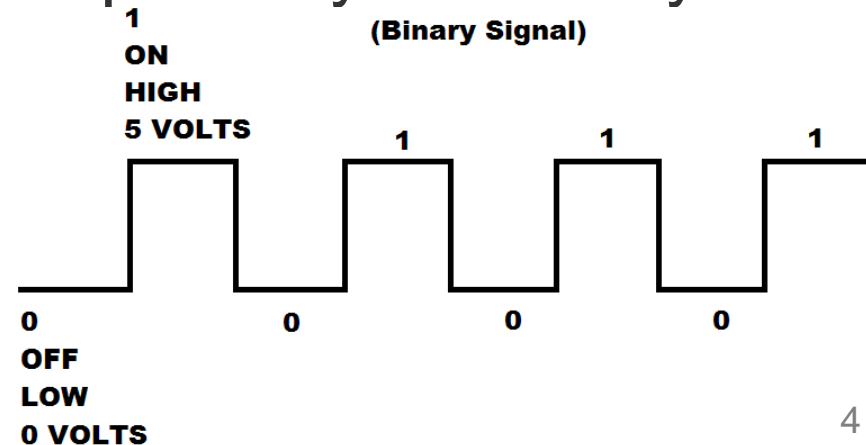
# Number Systems



- Common number systems:
  - The *radix* or *base* of the number system denotes the number of digits used in the system.

Binary ( <i>base 2</i> )	0 1
Octal ( <i>base 8</i> )	0 1 2 3 4 5 6 7
Decimal ( <i>base 10</i> )	0 1 2 3 4 5 6 7 8 9
Hexadecimal ( <i>base 16</i> )	0 1 2 3 4 5 6 7 8 9 A B C D E F

- The most natural way in a computer system is by **binary numbers (0, 1)**.
  - (0, 1) can be represented as (off, on) electrical signals.





- Number Representation
  - Number Systems
  - Integers
    - Unsigned Integer
    - Signed Integer
    - Arithmetic Operations
  - Floating-Point Numbers
    - Unsigned Binary Fraction
    - Floating-Point Number Representation
    - Arithmetic Operations
- Character Representation
  - ASCII

# Count to 100 in Decimal!



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17  
18 19 20 21 22 23 24 25 26 27 28 29  
30 31 32 33 34 35 36 37 38 39 40 41

The **Count to 100** by Ones Song

66 67 68 69 70 71 72 73 74 75 76 77  
78 79 80 81 82 83 84 85 86 87 88 89  
90 91 92 93 94 95 96 97 98 99 100

$$100 = 1 \times 10^2 + 0 \times 10^1 + 0 \times 10^0$$

# “Unsigned” Integer Representation



- Consider an  $n$ -bit (or  $n$ -digit) vector

$$B = (b_{n-1} \dots b_1 b_0)_2,$$

Denoting the base as a **subscript**

where  $b_i = 0$  or  $1$  (binary number) for  $0 \leq i \leq n - 1$

- **Most Significant Bit (MSB)**:  $b_{n-1}$  (i.e., the leftmost bit)
  - **Least Significant Bit (LSB)**:  $b_0$  (i.e., the rightmost bit)
- This vector can represent the decimal value for an unsigned integer  $V(B)$  in the range  $0$  to  $2^n - 1$ , where

$$V(B) = b_{n-1} \times 2^{n-1} + \dots + b_1 \times 2^1 + b_0 \times 2^0$$

- For example, if  $B = (1001)_2$ , where  $n = 4$

$$V(B) = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (9)_{10}$$

# Conversion of Number Systems



( Decimal ) <sub>10</sub>	( Binary ) <sub>2</sub>	( Octal ) <sub>8</sub>	( Hexadecimal ) <sub>16</sub>
( 0 0 ) <sub>10</sub>	( 0 0 0 0 ) <sub>2</sub>	( 0 0 ) <sub>8</sub>	( 0 ) <sub>16</sub>
( 0 1 ) <sub>10</sub>	( 0 0 0 1 ) <sub>2</sub>	( 0 1 ) <sub>8</sub>	( 1 ) <sub>16</sub>
( 0 2 ) <sub>10</sub>	( 0 0 1 0 ) <sub>2</sub>	( 0 2 ) <sub>8</sub>	( 2 ) <sub>16</sub>
( 0 3 ) <sub>10</sub>	( 0 0 1 1 ) <sub>2</sub>	( 0 3 ) <sub>8</sub>	( 3 ) <sub>16</sub>
( 0 4 ) <sub>10</sub>	( 0 1 0 0 ) <sub>2</sub>	( 0 4 ) <sub>8</sub>	( 4 ) <sub>16</sub>
( 0 5 ) <sub>10</sub>	( 0 1 0 1 ) <sub>2</sub>	( 0 5 ) <sub>8</sub>	( 5 ) <sub>16</sub>
( 0 6 ) <sub>10</sub>	( 0 1 1 0 ) <sub>2</sub>	( 0 6 ) <sub>8</sub>	( 6 ) <sub>16</sub>
( 0 7 ) <sub>10</sub>	( 0 1 1 1 ) <sub>2</sub>	( 0 7 ) <sub>8</sub>	( 7 ) <sub>16</sub>
( 0 8 ) <sub>10</sub>	( 1 0 0 0 ) <sub>2</sub>	( 1 0 ) <sub>8</sub>	( 8 ) <sub>16</sub>
( 0 9 ) <sub>10</sub>	( 1 0 0 1 ) <sub>2</sub>	( 1 1 ) <sub>8</sub>	( 9 ) <sub>16</sub>
( 1 0 ) <sub>10</sub>	( 1 0 1 0 ) <sub>2</sub>	( 1 2 ) <sub>8</sub>	( A ) <sub>16</sub>
( 1 1 ) <sub>10</sub>	( 1 0 1 1 ) <sub>2</sub>	( 1 3 ) <sub>8</sub>	( B ) <sub>16</sub>
( 1 2 ) <sub>10</sub>	( 1 1 0 0 ) <sub>2</sub>	( 1 4 ) <sub>8</sub>	( C ) <sub>16</sub>
( 1 3 ) <sub>10</sub>	( 1 1 0 1 ) <sub>2</sub>	( 1 5 ) <sub>8</sub>	( D ) <sub>16</sub>
( 1 4 ) <sub>10</sub>	( 1 1 1 0 ) <sub>2</sub>	( 1 6 ) <sub>8</sub>	( E ) <sub>16</sub>
( 1 5 ) <sub>10</sub>	( 1 1 1 1 ) <sub>2</sub>	( 1 7 ) <sub>8</sub>	( F ) <sub>16</sub>



# Class Exercise 2.1

Student ID: \_\_\_\_\_ Date: \_\_\_\_\_

Name: \_\_\_\_\_

- Represent  $(255)_{10}$  in binary, octal, and hexadecimal:

Binary ( <i>base 2</i> )	0 1
Octal ( <i>base 8</i> )	0 1 2 3 4 5 6 7
Decimal ( <i>base 10</i> )	0 1 2 3 4 5 6 7 8 9
Hexadecimal ( <i>base 16</i> )	0 1 2 3 4 5 6 7 8 9 A B C D E F

# “Signed” Integer Representation (1/3)



- To represent both **positive** and **negative** numbers, we need different systems to representing **signed integer**.
- In written decimal system, a signed integer is usually represented by a “+” or “-” sign and followed by the magnitude.
  - E.g. - 73, - 215, +349
- In **binary system**, we have three common systems:
  - ① **Sign-and-magnitude**
  - ② **1’s-complement**
  - ③ **2’s-complement**



# “Signed” Integer Representation (3/3)



B	Values Represented in Decimal		
$b_3b_2b_1b_0$	Sign-and-magnitude	1's-complement	2's-complement
0 1 1 1	+ 7	+ 7	+ 7
0 1 1 0	+ 6	+ 6	+ 6
0 1 0 1	+ 5	+ 5	+ 5
0 1 0 0	+ 4	+ 4	+ 4
0 0 1 1	+ 3	+ 3	+ 3
0 0 1 0	+ 2	+ 2	+ 2
0 0 0 1	+ 1	+ 1	+ 1
0 0 0 0	+ 0	+ 0	+ 0
1 0 0 0	- 0	- 7	- 8
1 0 0 1	- 1	- 6	- 7
1 0 1 0	- 2	- 5	- 6
1 0 1 1	- 3	- 4	- 5
1 1 0 0	- 4	- 3	- 4
1 1 0 1	- 5	- 2	- 3
1 1 1 0	- 6	- 1	- 2
1 1 1 1	- 7	- 0	- 1

# Class Exercise 2.2



- Question: Which representation system(s) uses distinct representations for  $+0$  and  $-0$  ?
- Answer: \_\_\_\_\_
  
- Question: Which representation system(s) has only one representation for  $0$  ?
- Answer: \_\_\_\_\_
  
- Question: Which representation system(s) is able to represent  $-8$  for 4-bit numbers?
- Answer: \_\_\_\_\_

# Class Exercise 2.3



- Question: Consider the decimal number  $-56$ . Please use 8 bits to represent it in:
  - Sign-and-magnitude: \_\_\_\_\_
  - 1's-complement: \_\_\_\_\_
  - 2's-complement: \_\_\_\_\_
- Question: Consider the 8-bit string  $10110101$ , what is its decimal value when interpreted as:
  - Sign-and-magnitude: \_\_\_\_\_
  - 1's-complement: \_\_\_\_\_
  - 2's-complement: \_\_\_\_\_
- Question: Given  $n$  bits, what is the range of integers can be represented by the three representations?
- Answer: \_\_\_\_\_

# Addition of “Unsigned” Integers



- Addition of 1-bit unsigned numbers:

$$\begin{array}{r} 0 \\ + 0 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \\ + 0 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 0 \\ + 1 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 1 \\ + 1 \\ \hline 10 \end{array}$$

carry-out sum

- To add multiple-bit numbers:

- We add bit pairs starting from the low-order (right) end, propagating carries toward the high-order (left) end.

- The **carry-out** from a bit pair becomes the **carry-in** to the next bit pair.
- The **carry-in** must be added to a bit pair in generating the sum and carry-out at that position.

- For example,

$$\begin{array}{r} \text{carry-in } 1 \\ \text{01111111} \\ + \text{00000001} \\ \hline \text{10000000} \end{array}$$

high-order ← ..... low-order

# Arithmetic of “Signed” Integers



- The three signed integer representation systems differ only in the way of representing **negative values**.
- Their relative merits on performing arithmetic operations can be summarized as follows:
  - **Sign-and-magnitude**: the **simplest representation**, but it is also **the most awkward** for addition/subtraction operations.
  - **1’s-complement**: **somewhat better** than the sign-and-magnitude system.
  - **2’s-complement**: **the most efficient** method for performing addition and subtraction operations.
    - This is also why the 2’s-complement system is the one **most often used** in modern computers.



# Why 2's-complement Arithmetic?



- First consider adding  $+7$  to  $-3$ :
  - What if we perform this addition by adding bit pairs from right to left (as what we did for  $n$ -bit unsigned numbers)?

$$\begin{array}{r} \text{leftmost} \\ \text{carry-out bit} \end{array} \begin{array}{r} 0 \ 1 \ 1 \ 1 \\ + \ 1 \ 1 \ 0 \ 1 \\ \hline 1 \ 0 \ 1 \ 0 \ 0 \end{array}$$

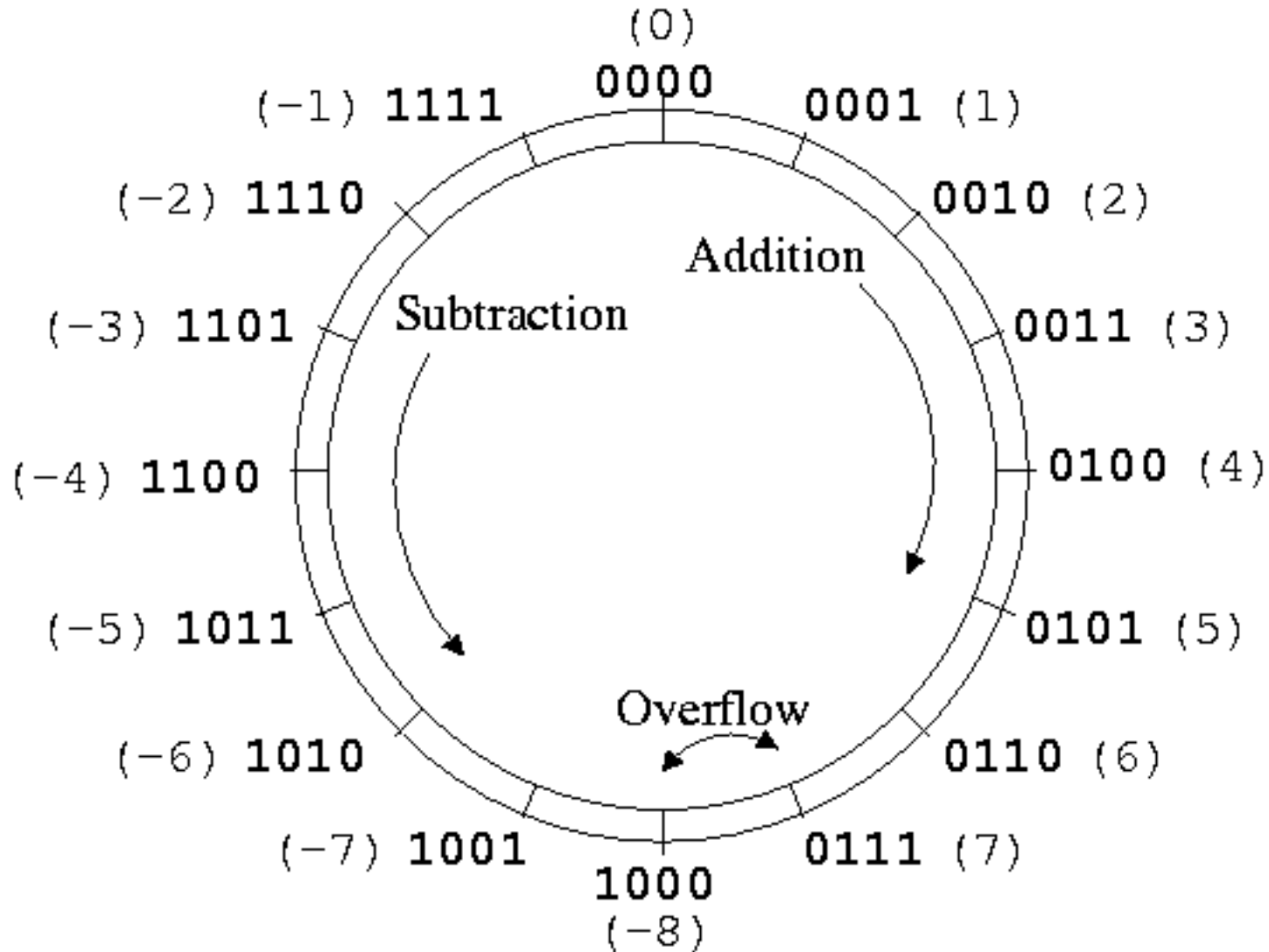
- If the **leftmost carry-out bit** is ignored, we get  $(+4)_{10}$ .
- Rules for  $n$ -bit signed number addition/subtraction:
  - $X + Y$ 
    - Add their  $n$ -bit 2's-complement representations from right to left
    - Ignore the carry-out bit at the MSB position
  - $X - Y$ 
    - Interpret as, and perform  $X + (-Y)$
  - *Note: The sum should be in the range of  $-2^{n-1} \sim (2^{n-1}-1)$*

# Class Exercise 2.4



- Using 4-bit 2's-complement number to calculate:
  - $2 + 3$
  - $4 + (-6)$
  - $(-5) + (-2)$
  - $2 - 4$
  - $(-7) - 1$
  - $(-7) - (-5)$

# 2's-Complement Number Wheel



# Overflow in Integer Arithmetic



- **Overflow:** The result of an arithmetic operation does not fall within the **representable range**.
  - In **Unsigned Number Arithmetic**:
    - **Rule:** A **carry-out of 1** from the MSB-bit always indicates an **overflow**.
      - E.g.  $(1111)_2 + (0001)_2 = (\underline{1} 0000)_2 \leftarrow \text{overflowed}$
      - E.g.  $(0111)_2 + (0001)_2 = (0 1000)_2 \leftarrow \text{no overflow}$
  - In **2's-complement Signed Number Arithmetic**:
    - The **carry-out bit** from the sign-bit is **not** an indicator of overflow.
      - E.g.  $(+7)_{10} + (+4)_{10} = (0111)_2 + (0100)_2 = (\underline{0} 1011)_2 = (-5)_{10}$
      - E.g.  $(-4)_{10} + (-6)_{10} = (1100)_2 + (1010)_2 = (\underline{1} 0110)_2 = (+6)_{10}$
    - **Observation:** Addition of opposite sign numbers never causes overflow.
      - E.g.  $(+7)_{10} + (-6)_{10} = (0111)_2 + (1010)_2 = (0001)_2 = (+1)_{10} \leftarrow \text{no overflow}$
    - **Rule:** If the two numbers are the same sign and the result is the opposite sign, we say that an **overflow** has occurred.
      - E.g.  $(+7)_{10} + (+4)_{10} = (0111)_2 + (0100)_2 = (1011)_2 = (-5)_{10} \leftarrow \text{overflowed}$
      - E.g.  $(-4)_{10} + (-6)_{10} = (1100)_2 + (1010)_2 = (0110)_2 = (+6)_{10} \leftarrow \text{overflowed}$

# Sign Extension



- We often need to represent a value given in a certain number of bits by using a **larger number of bits**.
  - That is, how to represent a signed integer by using a larger number of bits?
- **Sign Extension**: Simply repeat the “**sign bit**” as many times as needed to the left. (*Note: It can be applied to both 1’s and 2’s-complement, but **not** sign-and-magnitude*)
  - Positive Number: Add 0’s to the **left-hand-side**
    - E.g. **0**1111 → 0000 **0**1111
  - Negative Number: Add 1’s to the **left-hand-side**
    - E.g. **1**010 → 1111 **1**010

Example: Representing  $-2 \sim +1$  with 8 bits by 2’s-complement

B = $b_7b_6\dots b_0$	2’s complement
<u>000000</u> <b>0</b> 1	+1
<u>000000</u> <b>0</b> 0	+0
<u>111111</u> <b>1</b> 0	-2
<u>111111</u> <b>1</b> 1	-1



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# Unsigned Binary Fraction



- Consider a  $n$ -bit unsigned binary fraction:

$$B = (0.b_{-1}b_{-2} \dots b_{-n})_2$$

where  $b_{-i} = 0$  or  $1$  (**binary number**) for  $1 \leq i \leq n$

- This vector can represent the value for an unsigned binary fraction  $F(B)$ , where

$$F(B) = b_{-1} \times 2^{-1} + b_{-2} \times 2^{-2} + \dots + b_{-n} \times 2^{-n}$$

- The **range** of  $F(B)$  is

$$0 \leq F(B) \leq \underline{1 - 2^{-n}}$$

$$0 \leq F(B) \approx +1.0, \text{ for a large } n$$

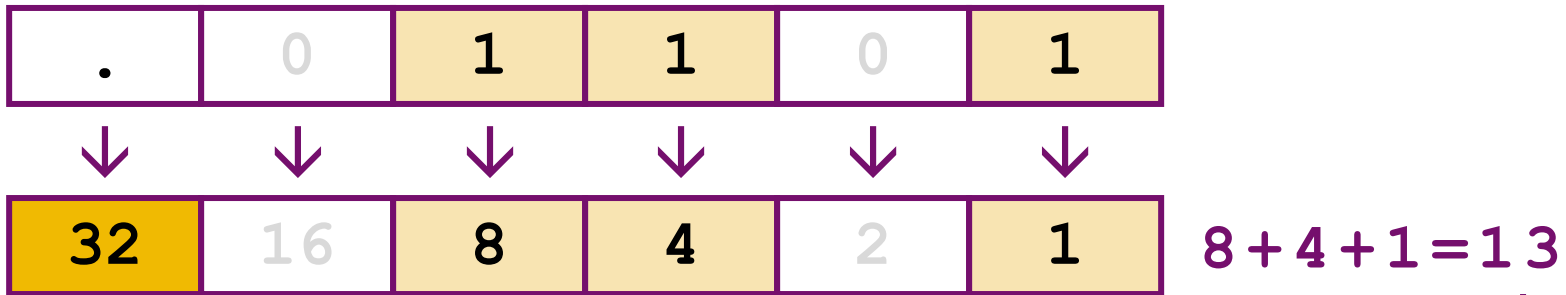
*Why? Geometric Series*

$$S_n = \sum_{i=1}^n a_i r^{i-1} = a_1 \left( \frac{1 - r^n}{1 - r} \right)$$

# Binary Fraction to Decimal Fraction



- What is the binary fraction  $(0.011010)_2$  in decimal ?



- Answer:  $13 / 32 = 0.40625$



# Decimal Fraction to Binary Fraction



- What is the decimal fraction  $(0.6875)_{10}$  in binary ?

$$\begin{array}{l} 0.6875 * 2 = \mathbf{1}.3750 \rightarrow 0.\mathbf{1}???\_2 \\ 0.\mathbf{3750} * 2 = \mathbf{0}.7500 \rightarrow 0.1\mathbf{0}??\_2 \\ 0.\mathbf{7500} * 2 = \mathbf{1}.5000 \rightarrow 0.10\mathbf{1}?\_2 \\ 0.\mathbf{5000} * 2 = \mathbf{1}.0000 \rightarrow 0.101\mathbf{1}\_2 \\ 0.\mathbf{0000} * 2 = 0 \rightarrow \mathbf{End} \end{array}$$

- Answer:  $(0.1011)_2$

*Why? Let's have an analogy in decimal:*

$$\begin{array}{l} 0.6875 * 10 = \mathbf{6}.875 \rightarrow (0.\mathbf{6}???)_{10} \\ 0.\mathbf{8750} * 10 = \mathbf{8}.7500 \rightarrow (0.6\mathbf{8}??)_{10} \end{array}$$

...

# Class Exercise 2.5



- What is the decimal fraction  $(0.1)_{10}$  in binary ?
- Answer:

# What did we learn so far?



- On one hand :
  - Some decimal fractions (e.g.  $(0.1)_{10}$ ) will produce **infinite binary fraction expansions**.
  - A  $n$ -bit unsigned fraction can only represent **values in the range of  $0 \sim 1 - 2^{-n}$**  and cannot represent **negative values**.
  - The position of the binary point in a floating-point number varies (that's way called **floating point!**).
$$0.232 * 10^4 = 2.320000 * 10^3 = 23.200000 * 10^2 = \dots$$
- On the other hand:
  - A  $n$ -bit signed integer in 2's-complement form can only represent **values in the range of  $-2^n \sim 2^n - 1$** .
- We need a **unique representation (form)** that can
  - ① Represent the **sign**, and the **position** of the floating point.
  - ② Represent both **very large** integers & **very small** fractions.

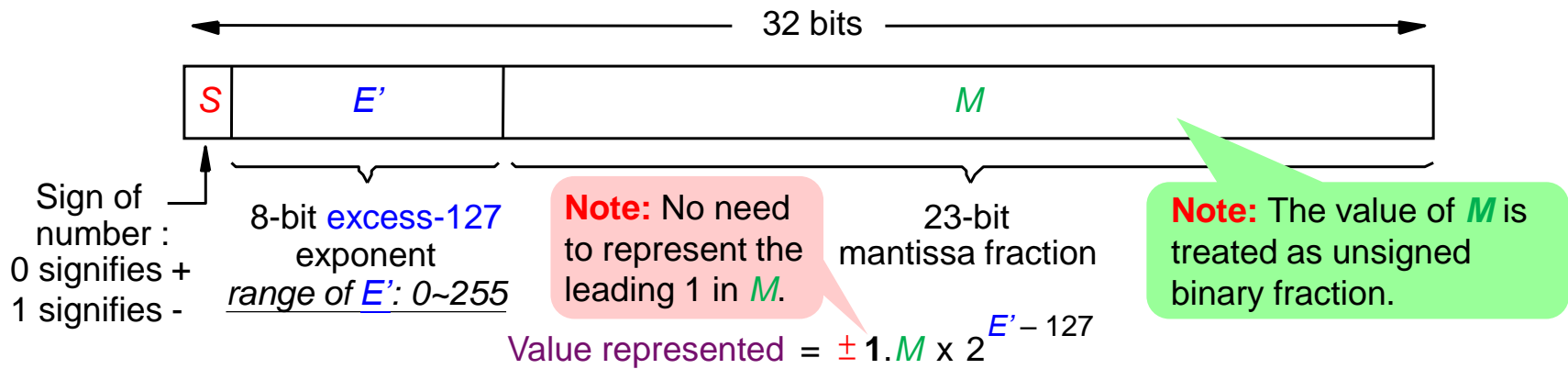
# Floating Point Number Representation

- In decimal **scientific notation**, numbers are written as :  
 $+6.0247 \times 10^{23}$ ,  $+3.7291 \times 10^{-27}$ ,  $-7.3000 \times 10^{-14}$ , ...
- The same approach can be used to represent binary floating-point numbers (using 2 as the base) by:
  - **Sign**: A sign for the number
  - **Mantissa**: Some significant bits
  - **Exponent**: A signed scale factor (implied base of 2)
- To have a **normalized representation** for floating-point numbers, we should normalize **Mantissa** in the range  $[1 \dots B)$ , where  $B$  is the base.
  - Binary System:  $[1 \dots 2)$ 
    - $(1.b_{-1}b_{-2}\dots b_{-n})_2$  must in the range of  $[1 \dots 2)$ .

# IEEE Standard 754 Single Precision



- The **single** precision format is a 32-bit representation.
  - The leftmost bit represents the sign, **S**, for the number.
  - The next 8 bits, **E'**, represent the **unsigned integer** for the *excess - 127 exponent* (with base of 2).
    - Note:** The actual **signed exponent E** is  $E' - 127$
  - The remaining 23 bits, **M**, are the significant bits.



$(00101000)_2 \rightarrow (40)_{10}$   
 $40 - 127 = -87$

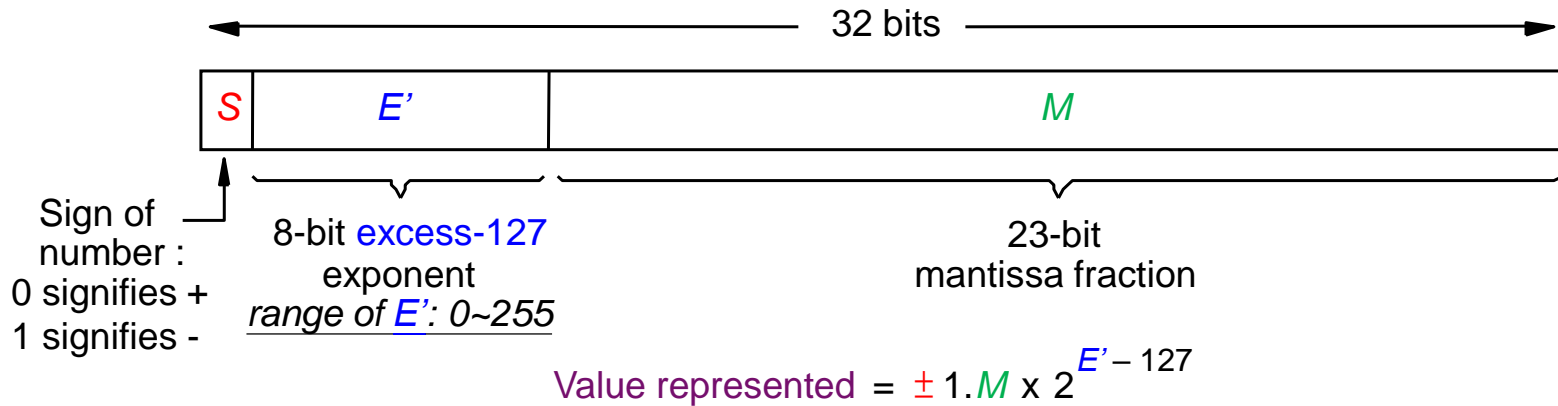
Value represented =  $+1.01101...0 \times 2^{-87}$

$(0.01101...0)_2 \rightarrow (0.40625)_{10}$

# Class Exercise 2.6



- What is the IEEE single precision number  $(40C0\ 0000)_{16}$  in decimal?



- Answer:

# Class Exercise 2.7



- What is  $(-0.5)_{10}$  in the IEEE single precision binary floating point format?
- Answer:



# Useful Tool

- IEEE-754 Floating Point Converter
  - <https://www.h-schmidt.net/FloatConverter/IEEE754.html>

**IEEE 754 Converter (JavaScript), V0.22**

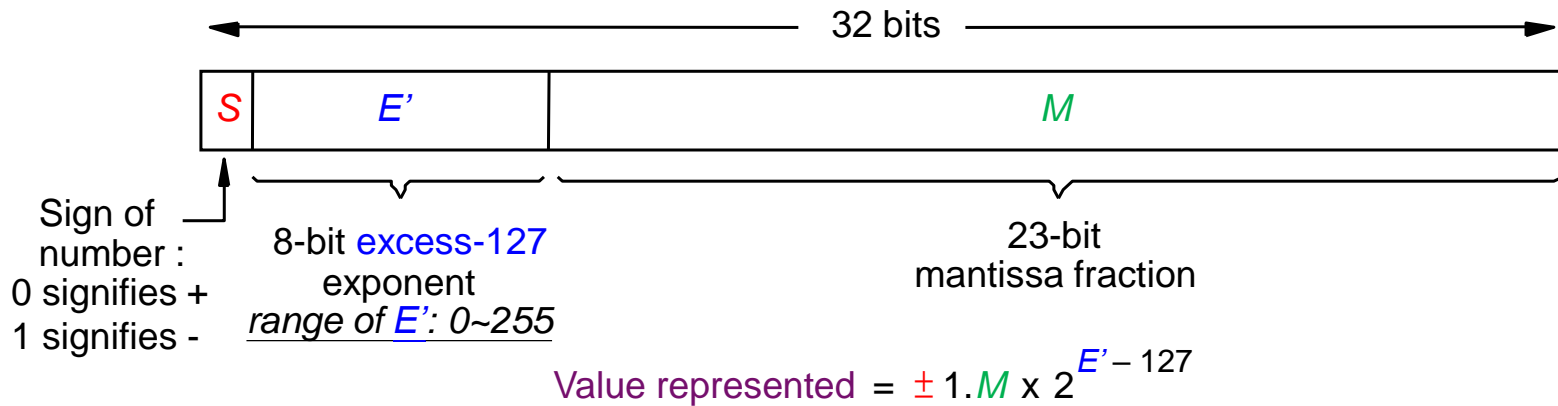
	Sign	Exponent	Mantissa
Value:	+1	$2^2$	1.5
Encoded as:	0	129	4194304
Binary:	<input type="checkbox"/>	<input checked="" type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input checked="" type="checkbox"/>	<input checked="" type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
Decimal representation	6.0		<input type="button" value="+1"/>
Value actually stored in float:	6		<input type="button" value="-1"/>
Error due to conversion:	<input type="text"/>		
Binary Representation	01000000110000000000000000000000		
Hexadecimal Representation	0x40c00000		

**IEEE 754 Converter (JavaScript), V0.22**

	Sign	Exponent	Mantissa
Value:	-1	$2^{-1}$	1.0
Encoded as:	1	126	0
Binary:	<input checked="" type="checkbox"/>	<input type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input checked="" type="checkbox"/> <input type="checkbox"/>	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>
You entered	-0.5		<input type="button" value="+1"/>
Value actually stored in float:	-0.5		<input type="button" value="-1"/>
Error due to conversion:	0.0		
Binary Representation	10111111000000000000000000000000		
Hexadecimal Representation	0xbf000000		



# Special Values

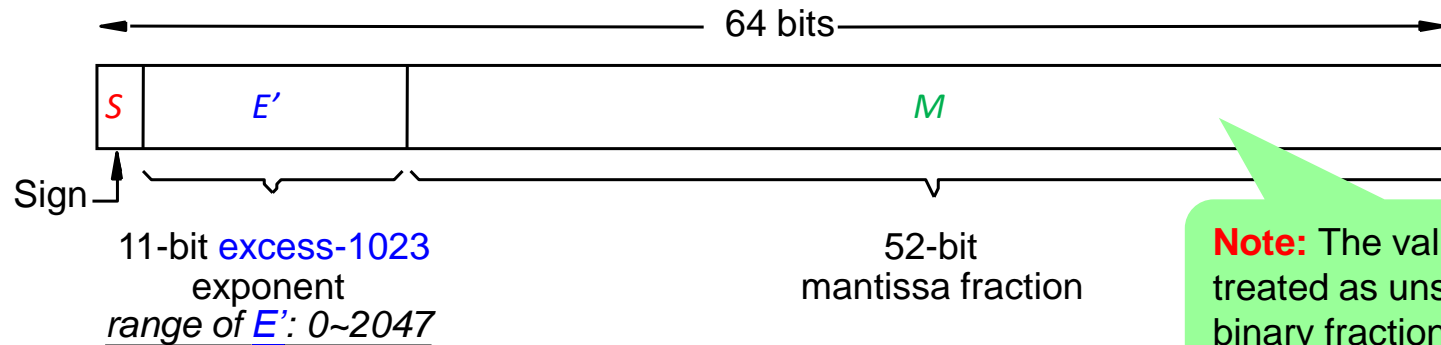


- When exponent  $E' = 0$  (all 0's) and mantissa  $M = 0$  :
  - The value 0 is represented.
- When exponent  $E' = 0$  (all 0's) and mantissa  $M \neq 0$  :
  - *Denormal values* (i.e. very small values) are represented.
- When exponent  $E' = 255$  (all 1's) and mantissa  $M = 0$  :
  - The value  $\infty$  is presented.
- When exponent  $E' = 255$  (all 1's) and mantissa  $M \neq 0$  :
  - *Not a Number (NaN)* (e.g.  $0/0$  or  $\sqrt{-1}$ ) is presented.

# IEEE Standard 754 Double Precision



- The **double** precision format is a 64-bit representation.
  - The leftmost bit represents the sign, **S**, for the number.
  - The next 11 bits, **E'**, represent the **unsigned integer** for the *excess-1023 exponent* (with base of 2).
    - **Note:** The actual **signed** exponent  $E$  is  $E' - 1023$ .
  - The remaining 52 bits, **M**, are the significant bits.



**Note:** The value of  $M$  is treated as unsigned binary fraction.

$$\text{Value represented} = \pm 1.M \times 2^{E' - 1023}$$

**Note:** No need to represent the leading 1 in  $M$ .

# Arithmetic on Floating-Point Number (1/2)

- When adding/subtracting floating-point numbers, their mantissas must be shifted with respect to each other.
  - E.g. adding  $(2.9400)_{10} \times 10^2$  to  $(4.3100)_{10} \times 10^4$ 
    - We rewrite  $(2.9400)_{10} \times 10^2$  as  $(0.0294)_{10} \times 10^4$
    - Then perform addition of the mantissas to get  $4.3394 \times 10^4$ .

## • Add/Subtract Rule

- 1) Choose the number with the smaller exponent and shift its mantissa right a number of steps equal to the difference in exponents.
- 2) Set the exponent of the result equal to the larger exponent.
- 3) Perform addition/subtraction on the mantissas and determine the sign of the result.
- 4) Normalize the resulting value, if necessary.

# Arithmetic on Floating-Point Number (2/2)

- Multiplication and division are somewhat easier than addition and subtraction.
  - No alignment of mantissas is needed.
- Multiply Rule
  - 1) Add the exponents and **subtract** 127 to maintain the excess-127 representation.
  - 2) **Multiply** the mantissas and determine the sign of the result.
  - 3) Normalize the resulting value, if necessary.
- Divide Rule
  - 1) Subtract the exponents and **add** 127 to maintain the excess-127 representation.
  - 2) **Divide** the mantissas and determine the sign of the result.
  - 3) Normalize the resulting value, if necessary.



- Number Representation
  - Number Systems
  - Integers
    - Unsigned Integer
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    - Arithmetic Operations
  - Floating-Point Numbers
    - Unsigned Binary Fraction
    - Floating-Point Number Representation
    - Arithmetic Operations
- **Character Representation**
  - **ASCII**

# Character Representation



- The most common encoding scheme for characters is **ASCII** (American Standard Code for Information Interchange).
- In ASCII encoding scheme, alphanumeric characters, operators, punctuation symbols, and control characters can be represented by 7-bit codes.
  - It is convenient to use an **8-bit *byte*** to represent a **character**.
    - The code occupies the low-order 7 bits with the high-order bit as 0.
- **Extended ASCII** encoding scheme uses **8-bit** (or even more) to represent the standard 7-bit ASCII characters, plus additional characters.

# ASCII Table



Dec	Bin	Hex	Char	Dec	Bin	Hex	Char	Dec	Bin	Hex	Char	Dec	Bin	Hex	Char
0	0000 0000	00	[NUL]	32	0010 0000	20	space	64	0100 0000	40	@	96	0110 0000	60	`
1	0000 0001	01	[SOH]	33	0010 0001	21	!	65	0100 0001	41	A	97	0110 0001	61	a
2	0000 0010	02	[STX]	34	0010 0010	22	"	66	0100 0010	42	B	98	0110 0010	62	b
3	0000 0011	03	[ETX]	35	0010 0011	23	#	67	0100 0011	43	C	99	0110 0011	63	c
4	0000 0100	04	[EOT]	36	0010 0100	24	\$	68	0100 0100	44	D	100	0110 0100	64	d
5	0000 0101	05	[ENQ]	37	0010 0101	25	%	69	0100 0101	45	E	101	0110 0101	65	e
6	0000 0110	06	[ACK]	38	0010 0110	26	&	70	0100 0110	46	F	102	0110 0110	66	f
7	0000 0111	07	[BEL]	39	0010 0111	27	'	71	0100 0111	47	G	103	0110 0111	67	g
8	0000 1000	08	[BS]	40	0010 1000	28	(	72	0100 1000	48	H	104	0110 1000	68	h
9	0000 1001	09	[TAB]	41	0010 1001	29	)	73	0100 1001	49	I	105	0110 1001	69	i
10	0000 1010	0A	[LF]	42	0010 1010	2A	*	74	0100 1010	4A	J	106	0110 1010	6A	j
11	0000 1011	0B	[VT]	43	0010 1011	2B	+	75	0100 1011	4B	K	107	0110 1011	6B	k
12	0000 1100	0C	[FF]	44	0010 1100	2C	,	76	0100 1100	4C	L	108	0110 1100	6C	l
13	0000 1101	0D	[CR]	45	0010 1101	2D	-	77	0100 1101	4D	M	109	0110 1101	6D	m
14	0000 1110	0E	[SO]	46	0010 1110	2E	.	78	0100 1110	4E	N	110	0110 1110	6E	n
15	0000 1111	0F	[SI]	47	0010 1111	2F	/	79	0100 1111	4F	O	111	0110 1111	6F	o
16	0001 0000	10	[DLE]	48	0011 0000	30	0	80	0101 0000	50	P	112	0111 0000	70	p
17	0001 0001	11	[DC1]	49	0011 0001	31	1	81	0101 0001	51	Q	113	0111 0001	71	q
18	0001 0010	12	[DC2]	50	0011 0010	32	2	82	0101 0010	52	R	114	0111 0010	72	r
19	0001 0011	13	[DC3]	51	0011 0011	33	3	83	0101 0011	53	S	115	0111 0011	73	s
20	0001 0100	14	[DC4]	52	0011 0100	34	4	84	0101 0100	54	T	116	0111 0100	74	t
21	0001 0101	15	[NAK]	53	0011 0101	35	5	85	0101 0101	55	U	117	0111 0101	75	u
22	0001 0110	16	[SYN]	54	0011 0110	36	6	86	0101 0110	56	V	118	0111 0110	76	v
23	0001 0111	17	[ETB]	55	0011 0111	37	7	87	0101 0111	57	W	119	0111 0111	77	w
24	0001 1000	18	[CAN]	56	0011 1000	38	8	88	0101 1000	58	X	120	0111 1000	78	x
25	0001 1001	19	[EM]	57	0011 1001	39	9	89	0101 1001	59	Y	121	0111 1001	79	y
26	0001 1010	1A	[SUB]	58	0011 1010	3A	:	90	0101 1010	5A	Z	122	0111 1010	7A	z
27	0001 1011	1B	[ESC]	59	0011 1011	3B	;	91	0101 1011	5B	[	123	0111 1011	7B	{
28	0001 1100	1C	[FS]	60	0011 1100	3C	<	92	0101 1100	5C	\	124	0111 1100	7C	
29	0001 1101	1D	[GS]	61	0011 1101	3D	=	93	0101 1101	5D	]	125	0111 1101	7D	}
30	0001 1110	1E	[RS]	62	0011 1110	3E	>	94	0101 1110	5E	^	126	0111 1110	7E	~
31	0001 1111	1F	[US]	63	0011 1111	3F	?	95	0101 1111	5F	_	127	0111 1111	7F	[DEL]

# Extended ASCII Table



ASCII control characters			ASCII printable characters			Extended ASCII characters										
00	NULL	(Null character)	32	space	64	@	96	`	128	Ç	160	á	192	Ł	224	Ó
01	SOH	(Start of Header)	33	!	65	A	97	a	129	ü	161	í	193	ł	225	ô
02	STX	(Start of Text)	34	"	66	B	98	b	130	é	162	ó	194	ł	226	õ
03	ETX	(End of Text)	35	#	67	C	99	c	131	â	163	ú	195	ł	227	ö
04	EOT	(End of Trans.)	36	\$	68	D	100	d	132	ä	164	ñ	196	—	228	ø
05	ENQ	(Enquiry)	37	%	69	E	101	e	133	à	165	Ñ	197	+	229	õ
06	ACK	(Acknowledgement)	38	&	70	F	102	f	134	â	166	ª	198	ã	230	µ
07	BEL	(Bell)	39	'	71	G	103	g	135	ç	167	º	199	Ä	231	þ
08	BS	(Backspace)	40	(	72	H	104	h	136	ê	168	¿	200	ℒ	232	ƒ
09	HT	(Horizontal Tab)	41	)	73	I	105	i	137	ë	169	®	201	ℓ	233	ú
10	LF	(Line feed)	42	*	74	J	106	j	138	è	170	™	202	ℓ	234	Û
11	VT	(Vertical Tab)	43	+	75	K	107	k	139	ï	171	½	203	ℓ	235	Ü
12	FF	(Form feed)	44	,	76	L	108	l	140	î	172	¼	204	ℓ	236	Ý
13	CR	(Carriage return)	45	-	77	M	109	m	141	ì	173	⅓	205	=	237	Ÿ
14	SO	(Shift Out)	46	.	78	N	110	n	142	Ë	174	¼	206	≠	238	—
15	SI	(Shift In)	47	/	79	O	111	o	143	Ä	175	½	207	≠	239	·
16	DLE	(Data link escape)	48	0	80	P	112	p	144	É	176	¾	208	ø	240	≡
17	DC1	(Device control 1)	49	1	81	Q	113	q	145	æ	177	¾	209	Ð	241	±
18	DC2	(Device control 2)	50	2	82	R	114	r	146	Æ	178	¾	210	È	242	±
19	DC3	(Device control 3)	51	3	83	S	115	s	147	ø	179	¾	211	Ë	243	¾
20	DC4	(Device control 4)	52	4	84	T	116	t	148	ö	180	¾	212	È	244	¶
21	NAK	(Negative acknowledge)	53	5	85	U	117	u	149	ò	181	¾	213	Ë	245	§
22	SYN	(Synchronous idle)	54	6	86	V	118	v	150	ù	182	¾	214	Ë	246	÷
23	ETB	(End of trans. block)	55	7	87	W	119	w	151	û	183	¾	215	Ë	247	°
24	CAN	(Cancel)	56	8	88	X	120	x	152	ÿ	184	¾	216	Ë	248	°
25	EM	(End of medium)	57	9	89	Y	121	y	153	ÿ	185	¾	217	Ë	249	°
26	SUB	(Substitute)	58	:	90	Z	122	z	154	ÿ	186	¾	218	Ë	250	°
27	ESC	(Escape)	59	;	91	[	123	{	155	ø	187	¾	219	Ë	251	°
28	FS	(File separator)	60	<	92	\	124		156	£	188	¾	220	Ë	252	°
29	GS	(Group separator)	61	=	93	]	125	}	157	Ø	189	¾	221	Ë	253	°
30	RS	(Record separator)	62	>	94	^	126	~	158	x	190	¾	222	Ë	254	°
31	US	(Unit separator)	63	?	95	_			159	f	191	¾	223	Ë	255	nbsp



# Class Exercise 2.7



- Represent “Hello, CSCI2510” using ASCII code:

	Decimal	Binary
H		
e		
l		
l		
o		
,		
C		
S		
C		
I		
2		
5		
1		
0		



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